Chapter 5

Exponential and Logarithmic Functions

Chapter 5 Mixed Review Worksheets

1. $f(x)=4-x \qquad g(x)=1+x^2$

a. $(f\circ g)(2)=f(g(2))$
$=f\left(1+2^2\right)$
$=f(5)$
$=4-5$
$=-1$

b. $(g\circ f)(-2)=g(f(-2))$
$=g\left(4-(-2)\right)$
$=g(6)$
$=1+6^2$
$=37$

c. $(f\circ f)(4)=f(f(4))$
$=f\left(4-4\right)$
$=f(0)$
$=4-0$
$=4$

d. $(g\circ g)(-1)=g(g(-1))$
$=g\left(1+(-1)^2\right)$
$=g(2)$
$=1+2^2$
$=5$

2. $f(x)=1-3x^2 \qquad g(x)=\sqrt{4-x}$

a. $(f\circ g)(2)=f(g(2))$
$=f\left(\sqrt{4-2}\right)$
$=f\left(\sqrt{2}\right)$
$=1-3\left(\sqrt{2}\right)^2$
$=1-3\cdot 2$
$=-5$

b. $(g\circ f)(-2)=g(f(-2))$
$=g\left(1-3(-2)^2\right)$
$=g(-11)$
$=\sqrt{4-(-11)}$
$=\sqrt{15}$

c. $(f\circ f)(4)=f(f(4))$
$=f\left(1-3(4)^2\right)$
$=f\left(-47\right)$
$=1-3(-47)^2$
$=-6626$

d. $(g\circ g)(-1)=g(g(-1))$
$=g\left(\sqrt{4-(-1)}\right)$
$=g\left(\sqrt{5}\right)$
$=\sqrt{4-\sqrt{5}}$

3. $f(x)=\dfrac{2}{1+2x^2} \qquad g(x)=3x$

a. $(f\circ g)(2)=f(g(2))$
$=f\left(3(2)\right)$
$=f(6)$
$=\dfrac{2}{1+2(6)^2}=\dfrac{2}{73}$

b. $(g\circ f)(-2)=g(f(-2))$
$=g\left(\dfrac{2}{1+2(-2)^2}\right)$
$=g\left(\dfrac{2}{9}\right)$
$=3\left(\dfrac{2}{9}\right)=\dfrac{2}{3}$

c. $(f \circ f)(4) = f(f(4))$

$$= f\left(\frac{2}{1+2(4)^2}\right)$$
$$= f\left(\frac{2}{33}\right)$$
$$= \frac{2}{1+2\left(\frac{2}{33}\right)^2}$$
$$= \frac{2}{\left(\frac{1097}{1089}\right)}$$
$$= \frac{2178}{1097}$$

d. $(g \circ g)(-1) = g(g(-1))$

$$= g(3(-1))$$
$$= g(-3)$$
$$= 3(-3)$$
$$= -9$$

4. $f(x) = 2x - 1 \qquad g(x) = 2x + 1$

The domain of f is $\{x \mid x \text{ is any real number}\}$.

The domain of g is $\{x \mid x \text{ is any real number}\}$.

$(f \circ g)(x) = f(g(x))$
$= f(2x+1)$
$= 2(2x+1) - 1$
$= 4x + 2 - 1$
$= 4x + 1$

Domain: $\{x \mid x \text{ is any real number}\}$.

$(g \circ f)(x) = g(f(x))$
$= g(2x-1)$
$= 2(2x-1) + 1$
$= 4x - 2 + 1$
$= 4x - 1$

Domain: $\{x \mid x \text{ is any real number}\}$.

$(f \circ f)(x) = f(f(x))$
$= f(2x-1)$
$= 2(2x-1) - 1$
$= 4x - 2 - 1$
$= 4x - 3$

Domain: $\{x \mid x \text{ is any real number}\}$.

$(g \circ g)(x) = g(g(x))$
$= g(2x+1)$
$= 2(2x+1) + 1$
$= 4x + 2 + 1$
$= 4x + 3$

Domain: $\{x \mid x \text{ is any real number}\}$.

5. $f(x) = 3x^2 + x + 1 \qquad g(x) = |3x|$

The domain of f is $\{x \mid x \text{ is any real number}\}$.

The domain of g is $\{x \mid x \text{ is any real number}\}$.

$(f \circ g)(x) = f(g(x))$
$= f(|3x|)$
$= 3(|3x|)^2 + (|3x|) + 1$
$= 27x^2 + 3|x| + 1$

Domain: $\{x \mid x \text{ is any real number}\}$.

$(g \circ f)(x) = g(f(x))$
$= g(3x^2 + x + 1)$
$= |3(3x^2 + x + 1)|$
$= 3|3x^2 + x + 1|$

Domain: $\{x \mid x \text{ is any real number}\}$.

$(f \circ f)(x) = f(f(x))$
$= f(3x^2 + x + 1)$
$= 3(3x^2 + x + 1)^2 + (3x^2 + x + 1) + 1$
$= 3(9x^4 + 6x^3 + 7x^2 + 2x + 1) + 3x^2 + x + 1 + 1$
$= 27x^4 + 18x^3 + 24x^2 + 7x + 5$

Domain: $\{x \mid x \text{ is any real number}\}$.

$(g \circ g)(x) = g(g(x))$
$= g(|3x|)$
$= |3|3x||$
$= 9|x|$

Domain: $\{x \mid x \text{ is any real number}\}$.

6. $f(x) = \sqrt{x-3} \qquad g(x) = \frac{3}{x}$

The domain of f is $\{x \mid x \geq 3\}$.

The domain of g is $\{x \mid x \neq 0\}$.

$(f \circ g)(x) = f(g(x))$

$= f\left(\frac{3}{x}\right) = \sqrt{\frac{3}{x} - 3} = \sqrt{\frac{3-3x}{x}}$

To find the domain, we must find where $p(x) = \frac{3-3x}{x} > 0$. p is zero or undefined when $x = 1$ and $x = 0$

Interval	$(-\infty, 0)$	$(0,1)$	$(1, \infty)$
Test Value	-1	$\frac{1}{2}$	2
Value of p	-6	3	$-\frac{3}{2}$
Conclusion	negative	positive	negative

Domain $\{x \mid 0 < x \le 1\}$.

$(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x-3}\right) = \frac{3}{\sqrt{x-3}}$

To find the domain, solve $x - 3 > 0$

$x > 3$

Domain $\{x \mid x > 3\}$

$(f \circ f)(x) = f(f(x)) = f\left(\sqrt{x-3}\right) = \sqrt{\sqrt{x-3} - 3}$

To find the domain, solve $\sqrt{x-3} - 3 \ge 0$

$\sqrt{x-3} \ge 3$

$x - 3 \ge 9$

$x \ge 12$

Domain $\{x \mid x \ge 12\}$.

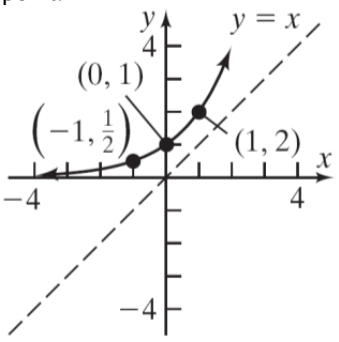
$(g \circ g)(x) = g(g(x)) = g\left(\frac{3}{x}\right) = \frac{3}{\left(\frac{3}{x}\right)} = 3\left(\frac{x}{3}\right) = x$

Domain $\{x \mid x \ne 0\}$.

7. **a.** The function is one-to-one because there are no two distinct inputs that correspond to the same output.

b. The inverse is $\{(4,-1),(2,0),(5,1),(7,3)\}$.

8. The function f is one-to-one because every horizontal line intersects the graph at exactly one point.



17. $f(x) = 4x + 3$

$y = 4x + 3$

$x = 4y + 3$ Inverse

$x - 3 = 4y$

$\frac{x-3}{4} = y$

$f^{-1}(x) = \frac{x-3}{4}$

Domain of f = Range of f^{-1}

= All real numbers.

Range of f = Domain of f^{-1}

= All real numbers.

10. $f(x) = \frac{2-x}{3+x}$

$y = \frac{2-x}{3+x}$

$x = \frac{2-y}{3+y}$ Inverse

$x(3+y) = 2-y$

$3x + xy = 2 - y$

$xy + y = 2 - 3x$

$y(x+1) = 2 - 3x$

$y = \frac{2-3x}{x+1}$

$f^{-1}(x) = \frac{2-3x}{x+1}$

Domain of f = Range of f^{-1}

= All real numbers except -3

Range of f = Domain of f^{-1}

= All real numbers except -1

11. $f(x) = \dfrac{3}{x^{1/3}}$

$$y = \frac{3}{x^{1/3}}$$

$$x = \frac{3}{y^{1/3}} \quad \text{Inverse}$$

$$xy^{1/3} = 3$$

$$y^{1/3} = \frac{3}{x}$$

$$y = \left(\frac{3}{x}\right)^3 = \frac{27}{x^3}$$

$$f^{-1}(x) = \frac{27}{x^3}$$

Domain of f = Range of f^{-1}
= All real numbers except 0

Range of f = Domain of f^{-1}
= All real numbers except 0